PHYSICAL REVIEW E VOLUME 58, NUMBER 4 OCTOBER 1998

Quenched disorder enhances chaotic diffusion

M. N. Popescu, Y. Braiman,* F. Family, and H. G. E. Hentschel
Department of Physics, Emory University, Atlanta, Georgia 30322

(Received 28 May 1998)

We show that chaotic diffusion of a single particle moving on a one-dimensional rough surface is enhanced by a small amount of spatial quenched disorder. In addition to enhanced diffusion we also find that there is a crossover from expanding to bounded motion. The crossover time to bounded motion decreases with increasing disorder, and there exists a threshold value of disorder above which chaotic motion is completely suppressed. [S1063-651X(98)50210-3]

PACS number(s): 81.40.Pq, 46.30.Pa, 05.45.+b

Understanding the effects of surface roughness on friction is a long standing and challenging problem [1,2]. For macroscopic systems, a rough interface is usually associated with an increase in friction due to asperities [2]. However, it has been recently suggested [3] that on an atomic scale this may not be always the case, and that friction can decrease with increased roughness of the surface. Motivated by this observation, in the present work we study the effects of substrate randomness on the chaotic diffusive motion of a particle in one dimension. Since the friction coefficient is related to the diffusion coefficient, studies on the effects of the randomness on chaotic diffusion will result in a better understanding of friction.

Diffusion is observed in many types of deterministic and stochastic systems that exhibit a wide variety of dynamical behaviors [4–13]. In deterministic chaotic systems, diffusion can be normal [4,13], with the mean-square displacement $\langle x^2\rangle$ proportional to time $t(\langle x^2\rangle \sim t)$, anomalous [5,7,12], with $\langle x^2\rangle \sim t^\gamma$, (enhanced for $\gamma>2$, dispersive for $\gamma<2$) or have a logarithmic time dependence ($\gamma=0$) [6,14].

Among the simplest dynamical systems in which chaotic diffusion can be observed are one-dimensional iterated maps of the form

$$x_{n+1}=x_n+F(x_n). \tag{1}$$

Studies of both normal and anomalous chaotic diffusion in such maps [with additional restrictions on $F(x)$ such as periodicity and reflection symmetry with respect to $x\to -x$] were motivated by the assumption that they capture the essential features of driven, damped diffusive motion in a periodic potential [4,7,12,14]. In many cases the simple form of these maps has made possible both large scale numerical simulations and analytical calculations of transport properties [4–8,12]. Little is known, however, about the effects of quenched spatial disorder on motion in otherwise periodic potentials.

In this work we report on an unusual effect that occurs in the case of continuous-time systems, namely, an increase in diffusion induced by the presence of a small amount of quenched disorder. For the parameter range considered, this phenomenon is only observed at low amounts of disorder, and above a certain threshold the chaotic diffusion is suppressed.

We consider the one-dimensional motion of a particle (in dimensionless units) on a disordered substrate:

$$\frac{d^2x}{dt^2}+\gamma\frac{dx}{dt}+\sin(x)=\Gamma\ \sin(\omega t)+\alpha\xi(x). \tag{2}$$

Here, γ is the damping coefficient, Γ and ω are, respectively, the amplitude and frequency of an oscillatory forcing, and $\alpha\xi(x)$ is the force due to the quenched disorder. For the present study, $\xi(x)\in[-1,1]$ are independent, uniformly distributed random variables with no spatial correlations corresponding to a piecewise constant force on the interval $[2k\pi,2(k+1)\pi)$, $k\in\mathbb{Z}$, and α is the amount of quenched disorder. Our interest in Eq. (2) is motivated by the fact that it can serve as a simplistic model for the systems studied in quartz microbalance experiments [15]. Clearly, the dynamics of a sliding monolayer is far more complicated than the dynamics of a single particle. However, for a weak substrate potential and at low coverages, the dynamics of a single particle can provide valuable insight into the motion of a monolayer on a rough surface. Indeed, phenomenological models [16,17] describing surface force apparatus experiments on confined liquids [18,19] have revealed important properties of the dynamics of the liquid into consideration.

It has recently been shown [13] that in the absence of quenched disorder ($\alpha=0$) normal diffusion is generated in both the chaotic and intermittent regimes of Eq. (2). The presence of quenched disorder ($\alpha\neq 0$) is assumed to introduce a more realistic representation of a substrate. Due to this spatial randomness, the periodicity and symmetry of the unperturbed potential are destroyed. Figure 1 shows a typical landscape of the resultant disordered potential,

$$\tilde{U}(x)=-\cos(x)-\alpha\int_0^x\xi(y)dy. \tag{3}$$

Since quenched disorder modifies the potential, it is natural to ask how it affects chaotic diffusion. The complex structure of the dynamical phase space corresponding to Eq. (2) in the absence of disorder [13,20] indicates the impor-

*Present address: Center for Engineering Science Advanced Research, Computer Science and Mathematics Division, Oak Ridge National Laboratory, P.O. Box 2008, Building 6010, MS-6355, Oak Ridge, TN 37831-6355.

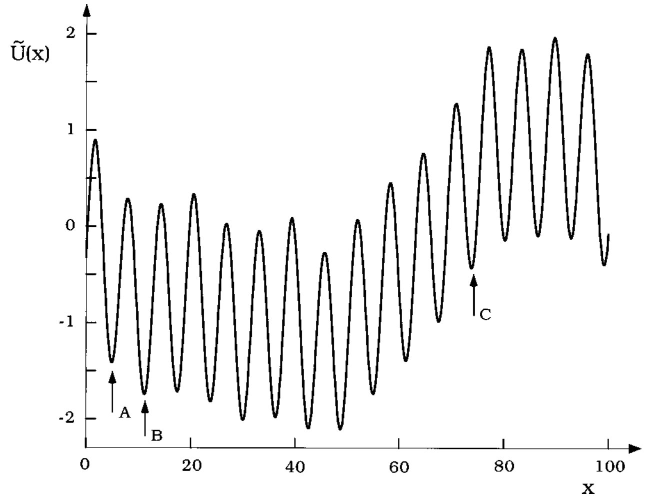

FIG. 1. Typical landscape of the disordered potential in Eq. (3) for $\alpha=10^{-1}$. Note that the motion toward right (left) is more favorable at *A* (*C*), while at *B* the potential barrier is almost the same for both directions.

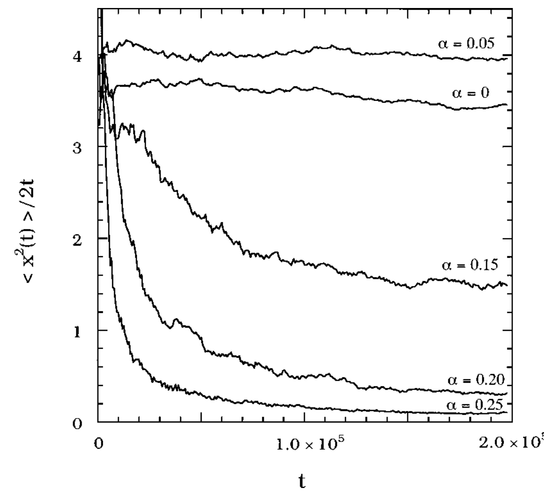

FIG. 3. Mean-square displacement as a function of time for several values of the amount of quenched disorder parameter α. The parameters used in Eq. (2) are $\gamma=0.2$, $\Gamma=1.2$, and $\omega=0.3$.

tance of properly choosing a region in parameter space. Thus, as a necessary first step, we have identified a set of parameters where, in the absence of disorder, the system shows chaotic diffusion. Specifically, we selected $\gamma=0.2$, $\Gamma=1.2$, and $\omega=0.3$. We note that simulations done for other sets of parameters (for example, the ones in Ref. [13]) have shown the same qualitative features.

Numerical solutions of Eq. (2) were obtained using a variable step Runge-Kutta-Fehlberg method [21]. Figure 2 shows typical results for the displacement $x(t)$ for the cases (a) $\alpha=0$ (no disorder), and (b) $\alpha=10^{-1}$. In the case of nonzero disorder we observe both trapped and expanding trajectories, depending on the initial conditions and on the specific realization of disorder; in the absence of disorder the trajectories are not trapped. This trapping of the trajectory inside a bounded region of phase space is one of the mecha-

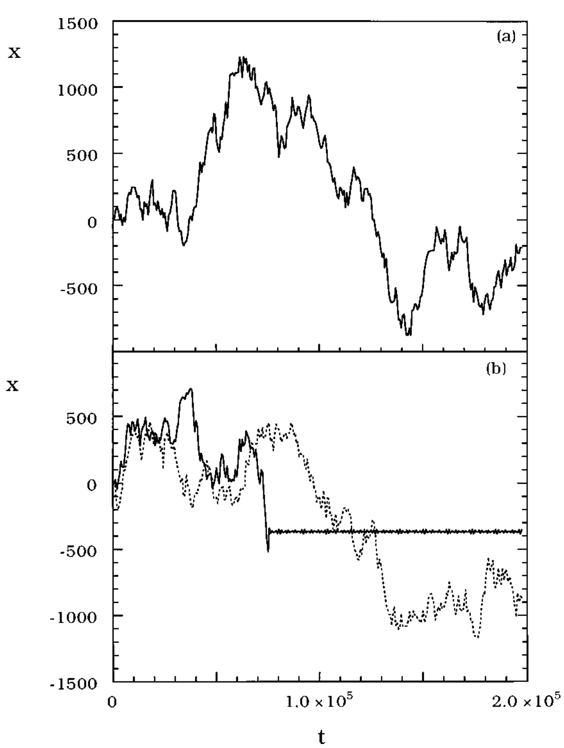

FIG. 2. Characteristic time series for the displacement of a diffusing particle described by Eq. (2) for $\gamma=0.2$, $\Gamma=1.2$, $\omega=0.3$, and (a) $\alpha=0$, (b) $\alpha=10^{-1}$. The two curves in (b) correspond to different initial conditions and different realizations of quenched disorder.

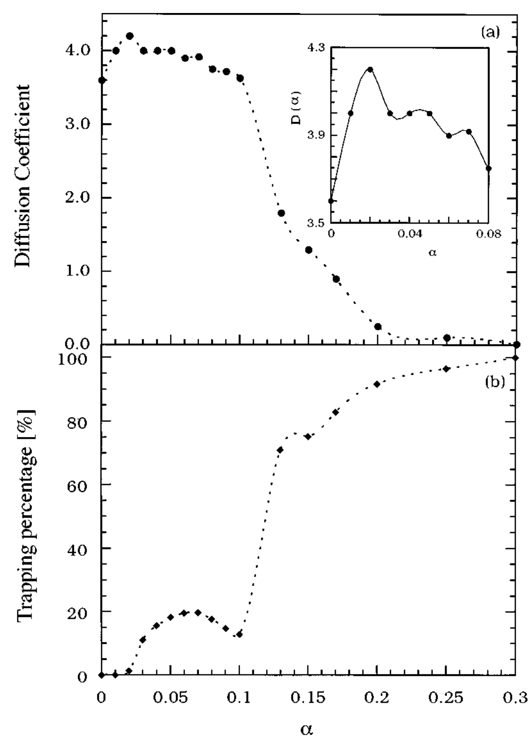

FIG. 4. (a) Diffusion coefficient $D(\alpha)$ as a function of the amount of quenched disorder; the points are results of the numerical simulations, the line is a cubic-spline interpolation to guide the eye. The inset shows the region where the diffusion coefficient is increased by disorder. (b) Percentage of trapped trajectories as a function of the amount of quenched disorder; the points are results of the numerical simulations, the line is a cubic-spline interpolation to guide the eye. The parameters are the same as in Fig. 2.

nisms through which quenched disorder affects the transport properties.

To calculate the mean-square displacement $\langle x^2(t)\rangle$, averages were performed over ensembles of 5000 trajectories starting from a very small cluster of different initial conditions centered around the origin $(x=0,\ \dot{x}=0)$ of the phase space. For each trajectory a different random sequence $\xi(x)$ was generated. In this way, the average over the ensemble of trajectories also includes an average over realizations of disorder. The ensemble described above was left to evolve for ~9400 external drive periods, while trajectories localized in a ''trap'' for a time longer than 750 external drive periods were not included in the averaging process. Therefore, the average mean-square displacement was computed only over the trajectories that were not trapped during the integration time period.

Figure 3 shows results for $\langle x^2(t)\rangle/2t$ as a function of the time t for several values of disorder parameter α. At long times the above ratio tends to a constant value, $D(\alpha)$, indicating that the quenched disorder does not change the normal character of the diffusion. We observe that $D(\alpha)$ is larger for $\alpha=0.05$ than for $\alpha=0$, i.e., small amounts of disorder increase the diffusion coefficient by approximately 15% [see inset in Fig. 4(a)]. Higher values of disorder, however, lead to a smaller $D(\alpha)$.

We have also followed the center of mass dynamics. For the entire range of realizations of the disorder considered, there was no translational motion of the center of mass. Although on short length-scales there was a net current induced by disorder, the large-scale average of this current is zero since the large-scale average of disorder is a ''flat'' substrate, and the transport is by simple diffusion.

Figure 4(a) shows the diffusion coefficients $D(\alpha) = \lim_{t\to\infty}(\langle x^2\rangle/2t)$ in the long-time limit as a function of the amount of quenched disorder. These results show that in the range $\alpha<0.1$ there is an increase in diffusion when compared to the diffusion on the periodic, unperturbed surface (see inset). At higher levels of disorder, the diffusion coefficient decreases toward zero. This decrease, correlated with the increase in the trapping probability shown in Fig. 4(b), indicates a crossover from expanding to bounded motion induced by the quenched disorder. The decrease in diffusion coefficient with increasing disorder is an expected result [8,9]. In our simulations, however, we first observe an increase of the diffusion coefficient with disorder, and only when the amount of disorder exceeds $\alpha=0.1$, the decrease of the diffusion coefficient is observed.

An analysis of the trajectories of Eq. (2) indicates that the spatial evolution of the statistical ensemble is similar to the case of Gaussian (stochastic) diffusion. We believe that in our simulations a small amount of quenched disorder can act as a source of thermal noise and increases the escape rates. In addition, introduction of a small amount of quenched disorder causes a spatial symmetry breaking that could also lead to an increase in the escape rates and, consequently, in the diffusion coefficient. This behavior resembles other known systems, such as thermal ratchets [9], kink diffusion [22], and stochastic diffusion with external linear bias [23], where disordered enhanced diffusion could possibly exist due to the spatial symmetry breaking. However, further analysis is needed for a better understanding of the mechanisms of the disorder enhanced diffusion.

In summary we have presented numerical evidence that the addition of small amounts of quenched disorder in the equation of motion of a continuous time system can induce an increase of the diffusion coefficient. We have shown that the presence of disorder does not change the character of normal diffusion and that the transport is diffusive. At high amounts of disorder the chaotic diffusion is suppressed, and almost all of the trajectories are localized.

We would like to thank Dr. Jacques G. Amar for his helpful comments and suggestions on the manuscript. This work was supported by grants from the U.S. Office of Naval Research and the National Science Foundation.

[1] B. N. J. Persson, *Sliding Friction: Physical Principles and Applications* (Springer-Verlag, Berlin, 1998).
[2] I. M. Hutchings, *Tribology* (Edward Arnold, London, 1992).
[3] Y. Braiman, F. Family, H. G. E. Hentschel, C. Mak, and J. Krim (unpublished).
[4] T. Geisel and J. Nierwertberg, Phys. Rev. Lett. **48**, 7 (1982).
[5] T. Geisel and S. Thomae, Phys. Rev. Lett. **52**, 1936 (1984).
[6] E. Marinari, G. Parisi, D. Ruelle, and P. Windey, Phys. Rev. Lett. **50**, 1223 (1983).
[7] T. Geisel, J. Nierwertberg, and A. Zacherl, Phys. Rev. Lett. **54**, 616 (1985).
[8] G. Radons, Phys. Rev. Lett. **77**, 4748 (1996).
[9] F. Marchesoni, Phys. Rev. E **56**, 2492 (1997).
[10] M. F. Shlesinger, G. Zaslavsky, and J. Klafter, Nature (London) **363**, 31 (1993).
[11] J. Klafter, M. F. Shlesinger, and G. Zumofen, Phys. Today **49** (2), 33 (1996).
[12] E. Barkai and J. Klafter, Phys. Rev. Lett. **79**, 2245 (1997).
[13] J. A. Blackburn and N. Grønbech-Jensen, Phys. Rev. E **53**, 3068 (1996).
[14] J. Krug and H. T. Dobbs, Phys. Rev. Lett. **76**, 4096 (1996).
[15] J. Krim, D. H. Solina, and R. Chiarello, Phys. Rev. Lett. **66**, 181 (1991); C. Daly and J. Krim, *ibid.* **76**, 803 (1996).
[16] M. G. Rozman, M. Urbakh, and J. Klafter, Phys. Rev. Lett. **77**, 683 (1996).
[17] J. M. Carlson and A. A. Batista, Phys. Rev. E **53**, 4153 (1996).
[18] H. Yoshizava, P. McGuiggan, and J. N. Israelachvili, Science **259**, 1305 (1993), H. Yoshizava and J. N. Israelachvili, J. Phys. Chem. **97**, 11 300 (1993).
[19] G. Reiter, A. L. Demirel, and S. Granick, Science **263**, 1741 (1994); A. L. Demirel and S. Granick, Phys. Rev. Lett. **77**, 4330 (1996).
[20] C. B. Whan and C. J. Lobb, Phys. Rev. E **53**, 405 (1996).
[21] W. H. Press, S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery, *Numerical Receipes in FORTRAN* (Cambridge University Press, Cambridge, 1995), pp. 704–716.
[22] O. M. Braun and Y. S. Kivshar, Phys. Rev. B **43**, 1060 (1991).
[23] R. Ghez, *A Primer of Diffusion Problems* (Wiley & Sons, New York, 1988), pp. 83–91.